A Globally Optimal, Dynamic Based, Operating Point Selection Scheme for MPC

Donald J. Chmielewski and Jui-Kun Peng

Department of Chemical & Environmental Engineering
Illinois Institute of Technology, Chicago, IL

Abstract: We propose a new formulation of the stochastically based minimally backed-off operating point (MBOP) selection problem. This scheme aims to combine the steady-state notions of profit with the dynamic, constraint observing notions of MPC design and tuning. The proposed formulation has a convex / reverse-convex form, and is readily solved globally via branch and bound. The formulation is trivially extended to the partial state information and discrete-time cases.

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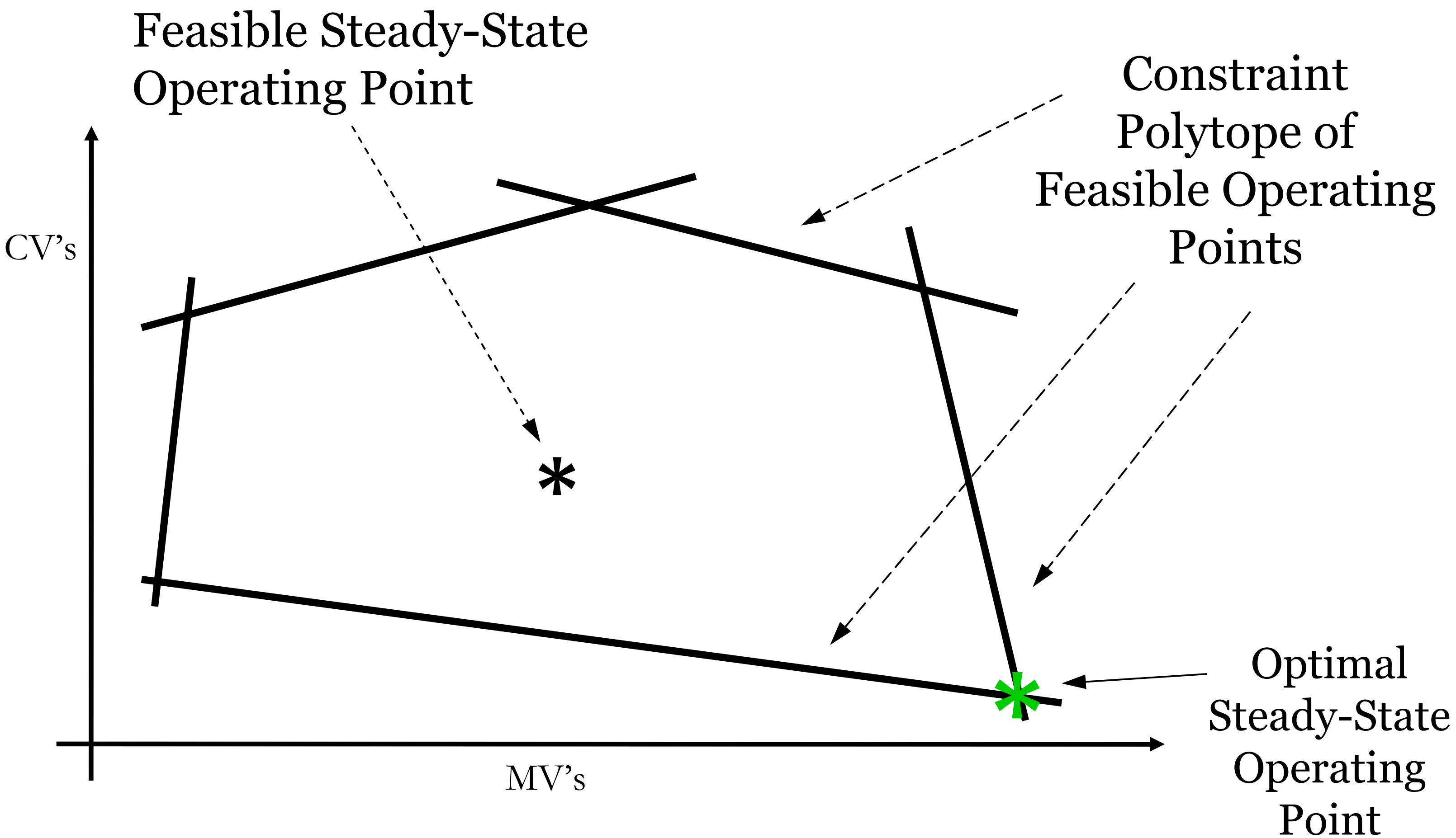
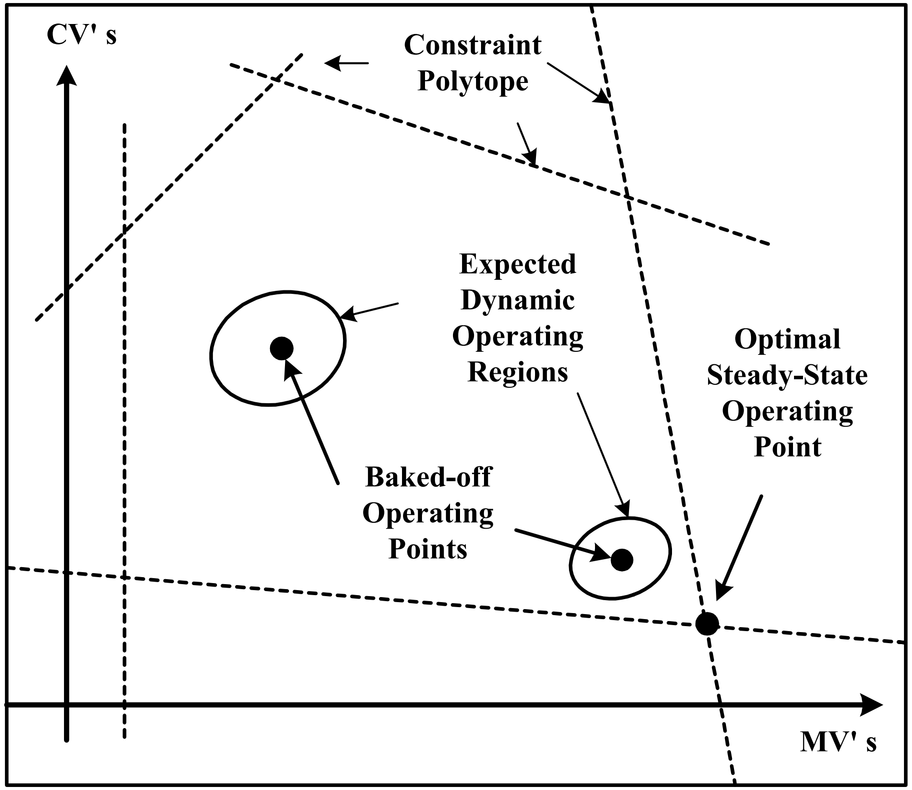
Real-Time Optimization

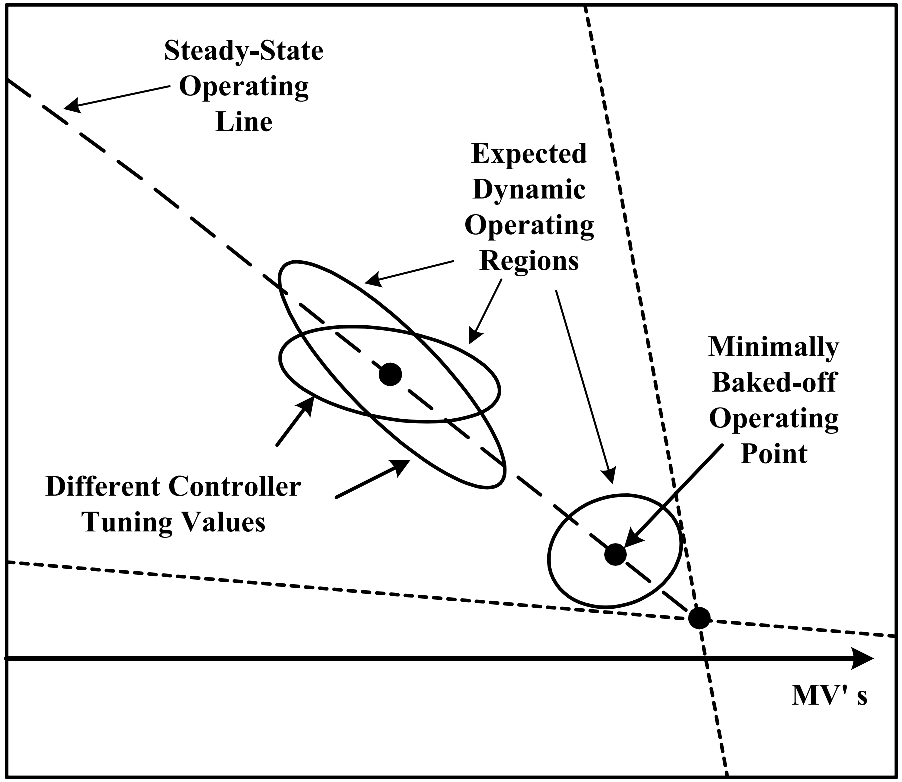


Minimally Backed-off Operating Point (MBOP) Selection

Goal: Bring the Backed-off Point as close as possible to the Optimal Steady-State.

Constraint: Do not allow the Expected Dynamic Operating Region outside the Constraint Polytope.

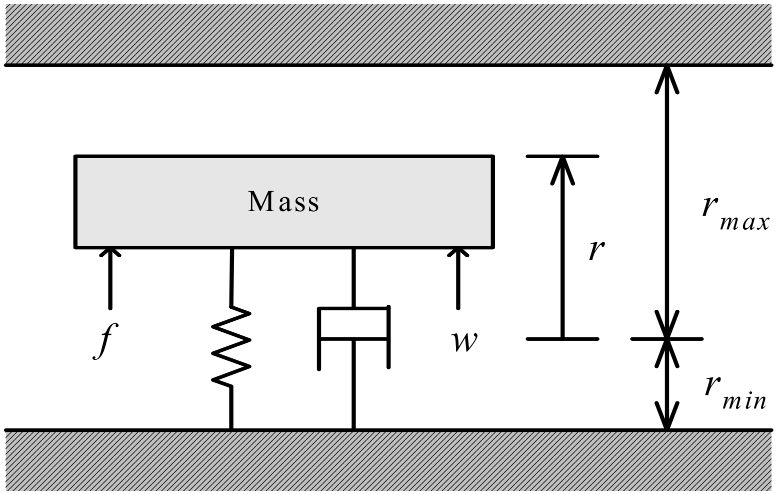




Steady-State Operating Line: Backed-off Points further limited by the Steady-State model.

Controller Tuning: Different tuning values will change the Size and Shape of the Expected Dynamic Operating Region.

Illustrative Example



System Model:

$$\begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$

where r is the mass position, v is the velocity,
f is the input force (MV) and
w is the disturbance force

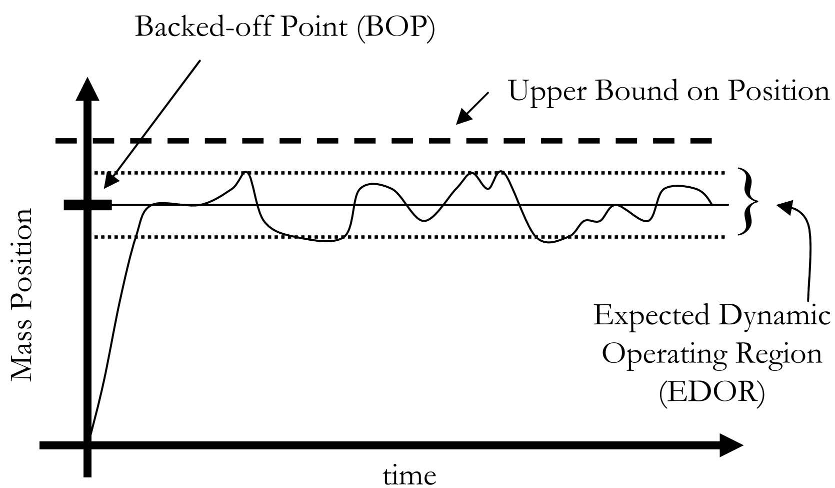
System Constraints:

$$-1 \leq r \leq 1 \quad \text{and} \quad 0 \leq f \leq 16$$

Design Objectives:

Steady-State: Put the average mass position as close as possible to the upper bound

Dynamic: In the face of disturbances, do not allow the mass position trajectory to extend beyond the upper bound.



Example Problem Formulation

$$\min_{\tilde{r},\tilde{f},x_{\min},x_{\max},u_{\min},u_{\max},\zeta_x,\zeta_u,L,\Sigma_x\geq 0} -\tilde{r}$$

$$s.t. \quad \tilde{f} = 3\tilde{r},$$

$$-2 \leq \tilde{r} \leq 0, \quad -12.8 \leq \tilde{f} \leq 2.2$$

$$x_{\min} = \tilde{r} + 2,\ x_{\max} = \tilde{r},$$

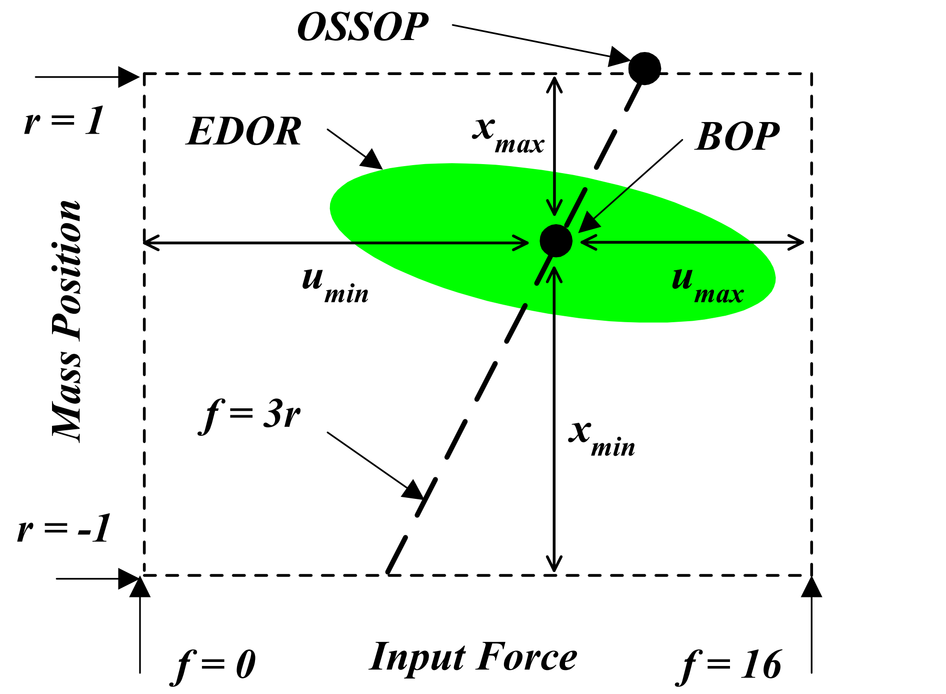
$$u_{\min} = \tilde{f} + 12.8,\ u_{\max} = \tilde{f} - 2.2,$$

$$\zeta_x < x_{\min}{}^2,\ \zeta_x < x_{\max}{}^2,$$

$$\zeta_u < u_{\min}{}^2,\ \zeta_u < u_{\max}{}^2,$$

$$\zeta_x = [1 \quad 0]\Sigma_x[1 \quad 0]^T,\ \zeta_u = L\Sigma_x L^T,$$

$$(A + BL)\Sigma_x + \Sigma_x(A + BL)^T + G\Sigma_w G^T = 0$$



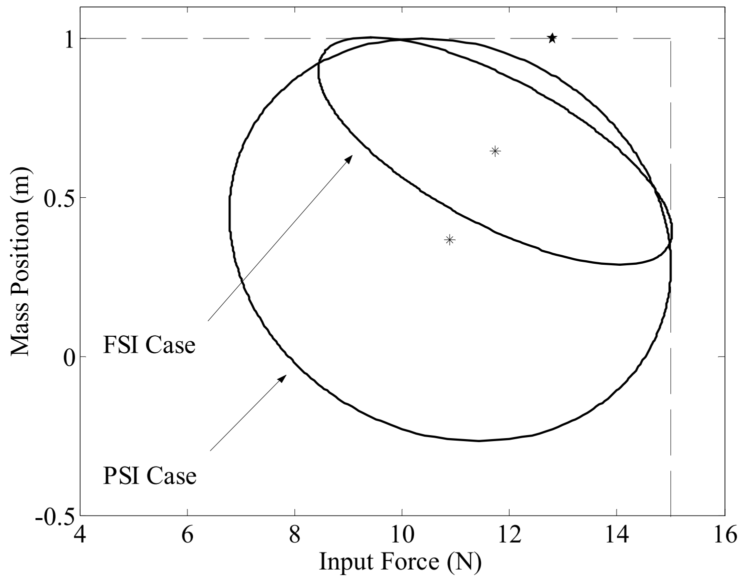
$\tilde{r}$ and $\tilde{f}$ are deviation variables w.r.t. OSSOP.

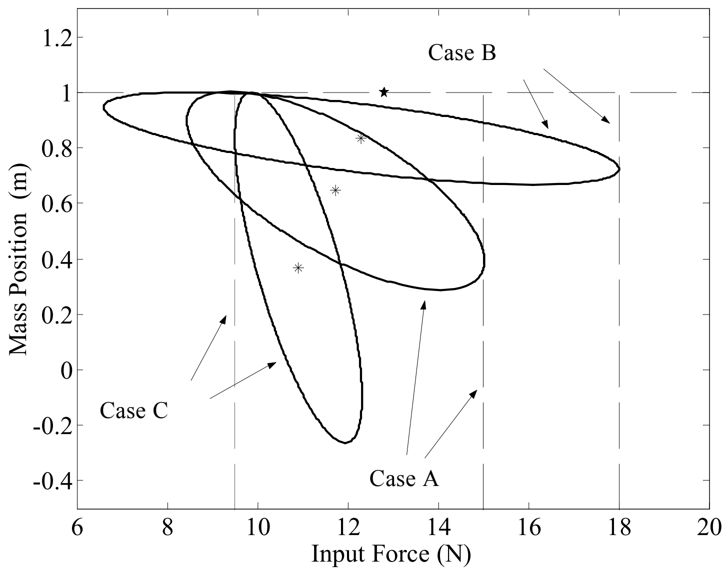
$x_{\min}$ is distance from BOP to constraint.

$2\sqrt{\zeta_x}$ is the EDOR height.

$\sqrt{\zeta_x} < x_{\max}$ guarantees EDOR within constraints.

Numeric Solutions





FSI Case: Full State Information.

Controller is $u(t) = Lx(t)$

PSI Case: Partial Information: 1 Velocity Sensor.

Controller is $u(t) = L\hat{x}(t)$

where $\hat{x}(t)$ is from a state estimator.

Case A: Same as FSI Case.

Case B: Same as Case A, but max force changed from 15 to 18.

Case C: Same as Case A, but min force changed from 0 to 9.5.

General MBOP Formulation

Dynamic System in Actual Variables:

$$\dot{s} = As + Bm + Gp, \quad z = D_x s + D_u m + D_w p, \quad d_{\min} \leq z_{ss} \leq d_{\max}$$

Controlled System in Deviation Variables:

$$\dot{x} = Ax + Bu + Gw, \quad u = Lx, \quad \text{Size of } w \text{ given by } \Sigma_w.$$

General Problem Formulation:

$$\min_{\tilde{s}_{ss}, \tilde{m}_{ss}, \tilde{z}_{ss}, \zeta, L, \Sigma_x \geq 0} \quad d_s^T \tilde{s}_{ss} + d_m^T \tilde{m}_{ss}$$

$$s.t. \quad 0 = A\tilde{s}_{ss} + B\tilde{m}_{ss}, \quad \tilde{z}_{ss} = D_x \tilde{s}_{ss} + D_u \tilde{m}_{ss}, \quad \tilde{d}_{\min} \leq \tilde{z}_{ss} \leq \tilde{d}_{\max}$$

$$\zeta_i < (\tilde{z}_{ss,i} - \tilde{d}_{\min,i})^2, \quad \zeta_i < (\tilde{z}_{ss,i} - \tilde{d}_{\max,i})^2, \qquad i = 1 \cdots n_z$$

$$\zeta_i = \phi_i [(D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T] \phi_i^T, \quad i = 1 \cdots n_z$$

$$0 = (A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T$$

Constraint Convexification Theorem

$$\exists \text{ stabilizing } L, \Sigma_x \geq 0, \text{ and } \zeta_i, \quad i = 1 \cdots n_z$$

$$s.t. \quad (A+BL)\Sigma_x + \Sigma_x (A+BL)^T + G\Sigma_w G^T = 0, \quad \zeta_i < \bar{z}_i^2, \quad i = 1 \cdots n_z$$

$$\text{and} \quad \zeta_i = \phi_i [(D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T] \phi_i^T, \quad i = 1 \cdots n_z$$

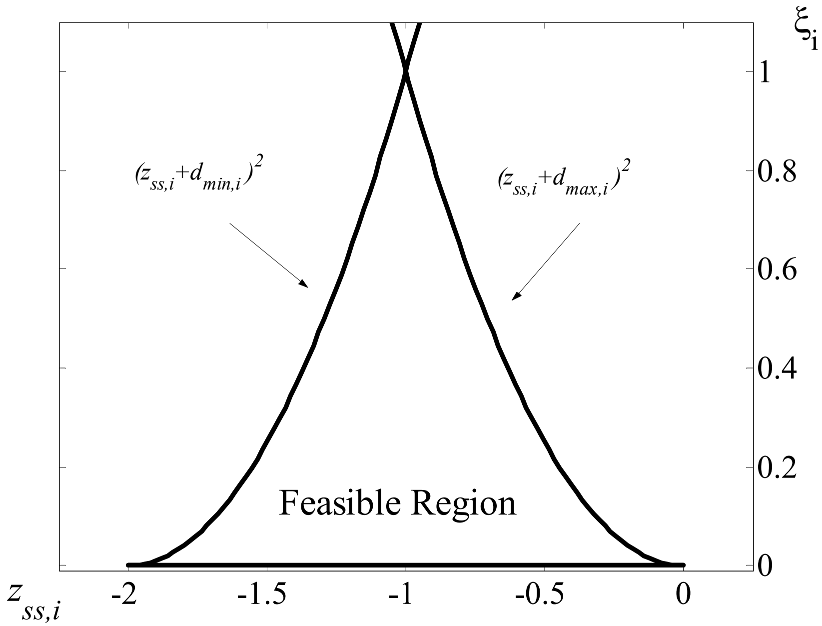
if and only if

$$\exists \quad L, X > 0 \text{ and } \zeta_i, \quad i = 1 \cdots n_z$$

$$s.t. \quad (AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0, \quad \zeta_i < \bar{z}_i^2, \quad i = 1 \cdots n_z$$

$$\text{and} \quad \begin{bmatrix} \zeta_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\ (D_x X + D_u Y)^T \phi_i^T & X \end{bmatrix} > 0 \quad , \quad i = 1 \cdots n_z$$

Reverse-Convex Constraints

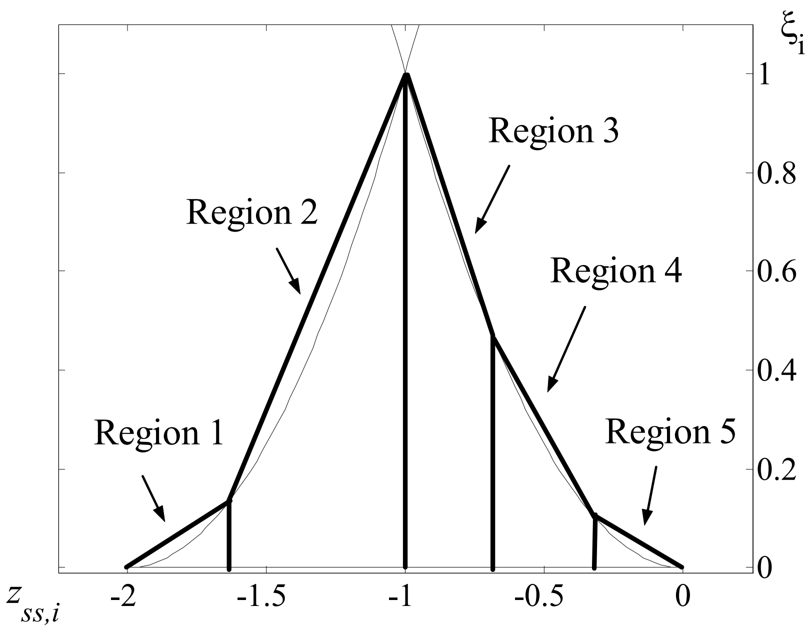


Reverse-Convex Constraints required to Guarantee EDOR within the Polytope:

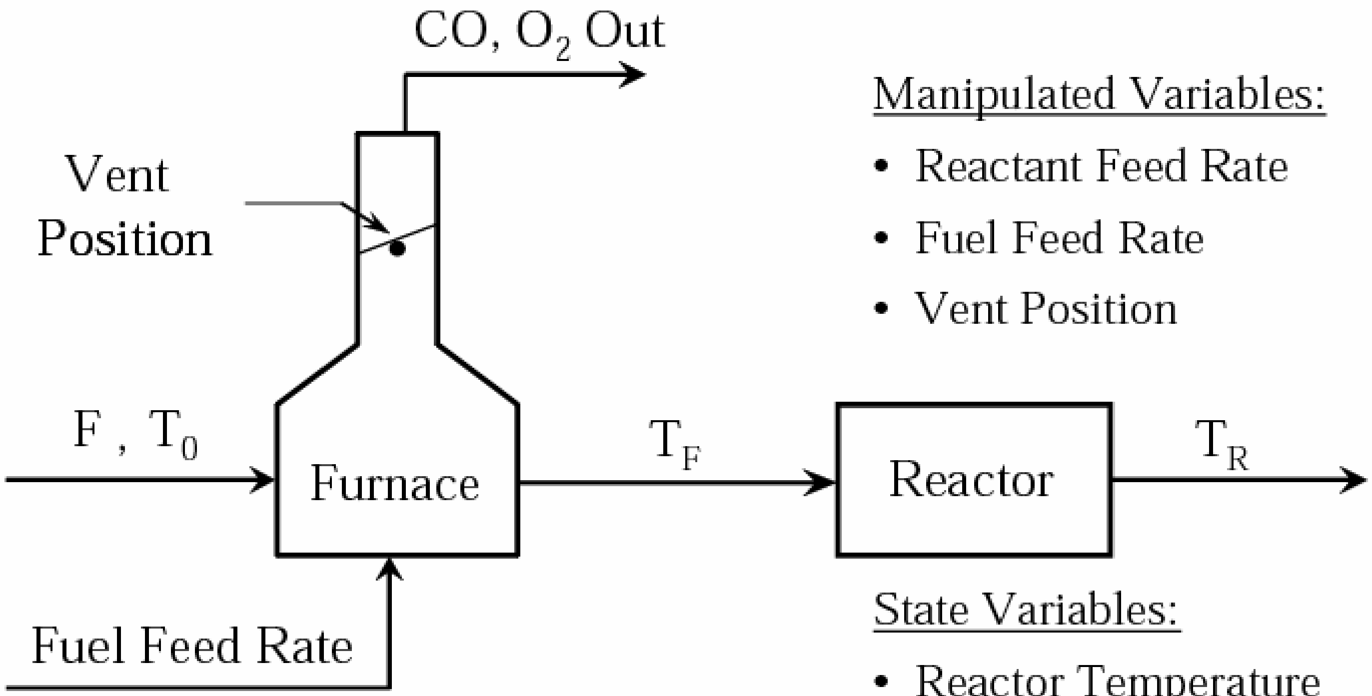
$$\zeta_i < (\tilde{z}_{ss,i} - \tilde{d}_{\min,i})^2$$

and $$\zeta_i < (\tilde{z}_{ss,i} - \tilde{d}_{\max,i})^2$$

Branch and Bound Algorithm used to find Globally Optimal Solutions



Reactor Furnace Example



<u>Manipulated Variables:</u>

- Reactant Feed Rate
- Fuel Feed Rate
- Vent Position

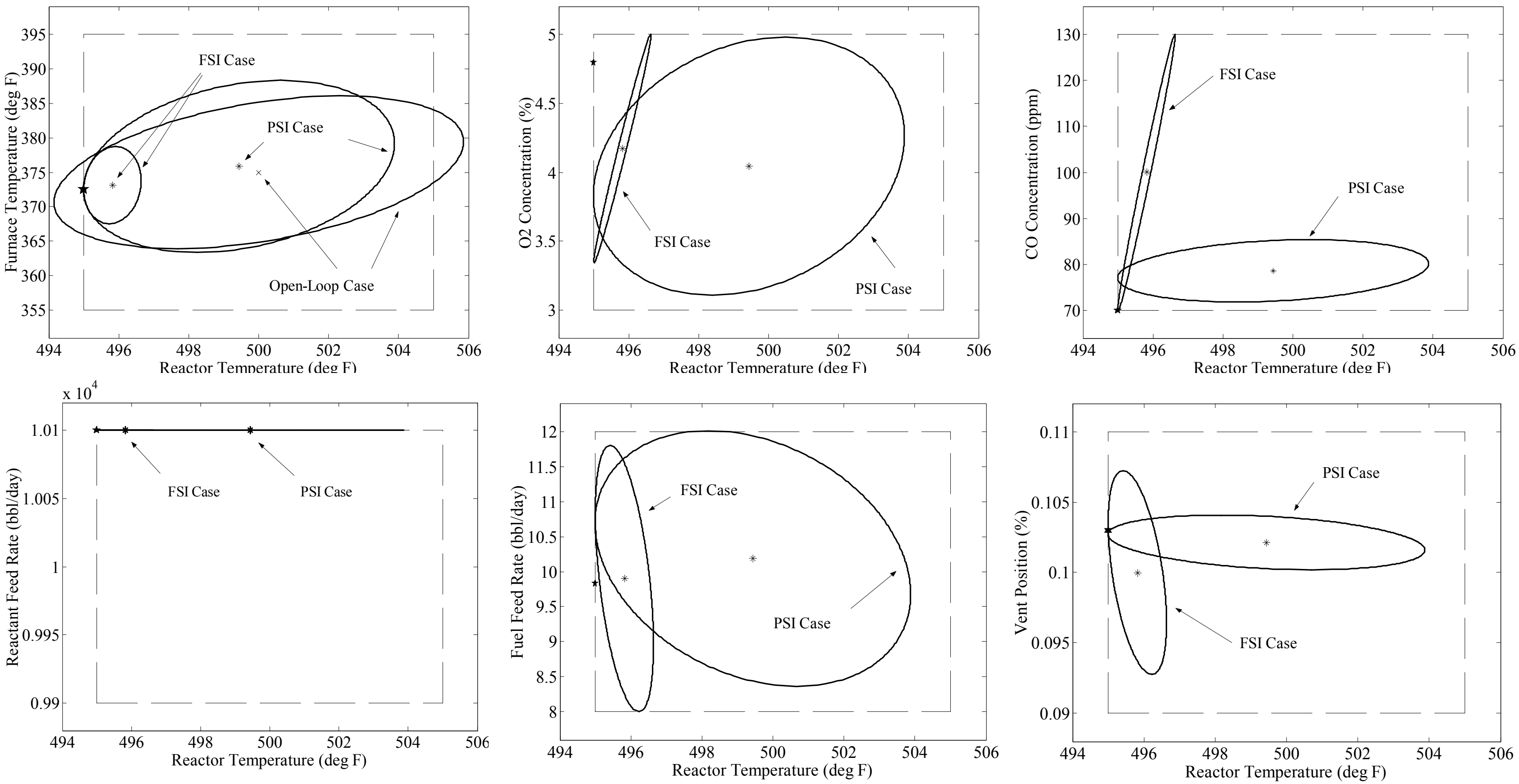
<u>State Variables:</u>

- Reactor Temperature
- Furnace Temperature
- Furnace O_2
- Furnace CO

<u>Disturbance Input:</u>

- Feed Temperature

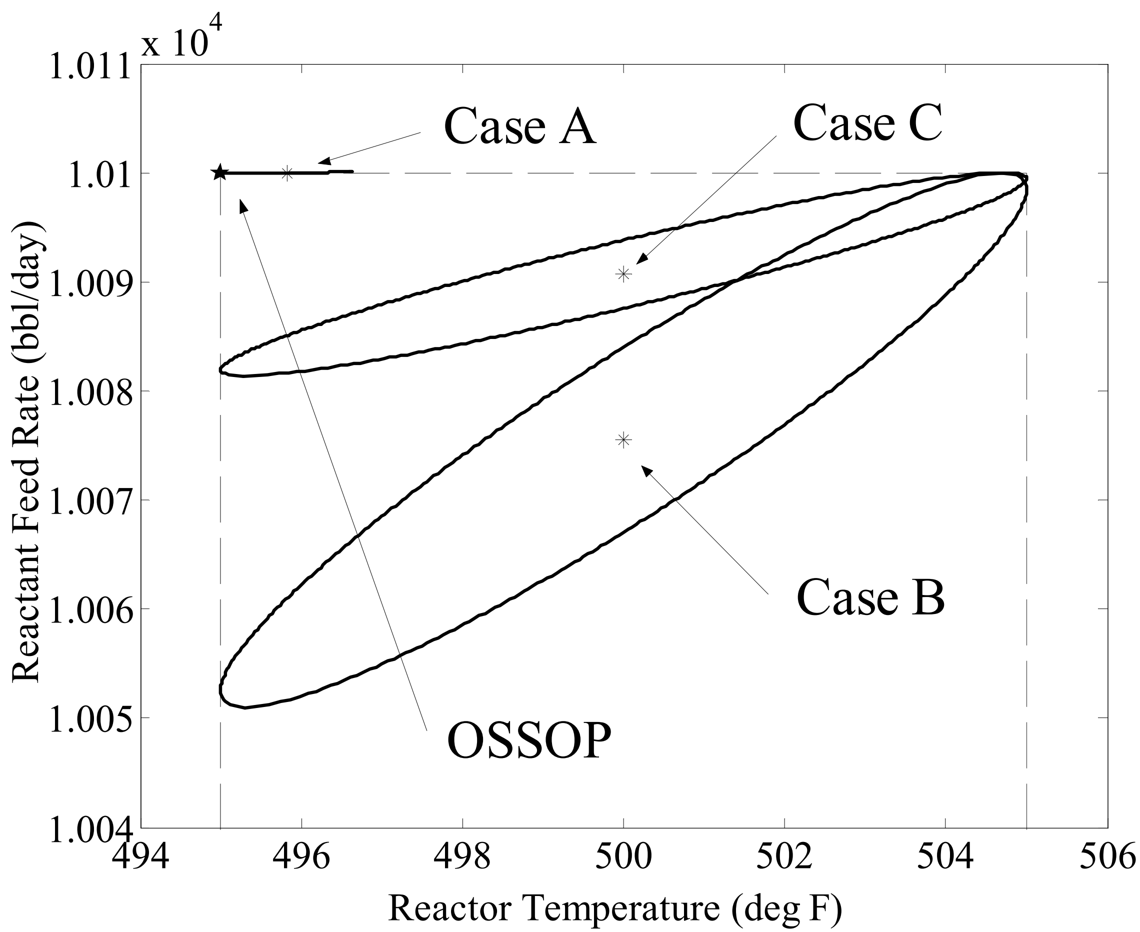
Numeric Solutions



Profit = − 0.01 C_{O2} + 10 F_{in} − 30 F_{fuel}

PSI case uses sensor at T_R

Comparison of Profits



OSSOP:

Profit = \$100,704

Case A: Same as FSI Case.

Profit = \$100,698

Case B: Same as Case A, but fuel feed bounds changed to 10±0.25.

Profit = \$100,449

Case C: Same as Case A, but O2 concentration bound changed to 4%.

Profit = \$100,599